Corrigé
Complexes
Changer de forme

Exercice 1. exponentielle vers algébrique

Passer les nombre complexes suivants sous la forme algébrique :

1. $e^{i\pi} = \cos(\pi) + \sin(\pi)i = -1 + 0i = -1$

2. $e^{5i\pi} = \cos(5\pi) + \sin(5\pi)i = -1 + 0i = -1$

3. $2e^{i\frac{\pi}{2}} = 2\left(\cos(\frac{\pi}{2}) + \sin(\frac{\pi}{2})i\right) = 2(0+i)$

4. $3e^{i\frac{\pi}{3}} = 3\left(\cos(\frac{\pi}{3}) + \sin(\frac{\pi}{3})i\right) = 3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

5. $4e^{i\frac{\pi}{4}} = 4\left(\cos(\frac{\pi}{4}) + \sin(\frac{\pi}{4})i\right) = 4\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

6. $5e^{i\frac{\pi}{6}} = 5\left(\cos(\frac{\pi}{6}) + \sin(\frac{\pi}{6})i\right) = 5\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

7. $\sqrt{2}e^{-i\frac{\pi}{2}} = \sqrt{2}\left(\cos(\frac{-\pi}{2}) + \sin(\frac{-\pi}{2})i\right) = \sqrt{2}(0-i)$

8. $\sqrt{2}e^{-i\frac{\pi}{3}} = \sqrt{2}\left(\cos(\frac{-\pi}{3}) + \sin(\frac{-\pi}{3})i\right) = \sqrt{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

9. $\sqrt{3}e^{-i\frac{\pi}{4}} = \sqrt{3}\left(\cos(\frac{-\pi}{4}) + \sin(\frac{-\pi}{4})i\right) = \sqrt{3}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

10. $\sqrt{3}e^{-i\frac{\pi}{6}} = \sqrt{3}\left(\cos(\frac{-\pi}{6}) + \sin(\frac{-\pi}{6})i\right) = \sqrt{3}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

11. $10e^{-i\frac{5\pi}{2}} = 10\left(\cos(-\frac{5\pi}{2}) + \sin(-\frac{5\pi}{2})i\right) = 10(0-i)$

12. $20e^{i\frac{2\pi}{3}} = 20\left(\cos(\frac{2\pi}{3}) + \sin(\frac{2\pi}{3})i\right) = 20\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)$

13. $30e^{-i\frac{3\pi}{4}} = 30\left(\cos(\frac{-3\pi}{4}) + \sin(\frac{-3\pi}{4})i\right) = 30\left(\frac{-\sqrt{2}}{2} + \frac{-\sqrt{2}}{2}i\right)$

14. $40e^{i\frac{7\pi}{6}} = 40\left(\cos(\frac{7\pi}{6}) + \sin(\frac{7\pi}{6})i\right) = 40\left(\frac{-\sqrt{3}}{2} + \frac{-1}{2}i\right)$

15. $e^{i\times 0} = \cos(0) + \sin(0)i = 1 + 0i = 1$

Exercice 2. algébrique vers exponentielle

Passer les nombre complexes suivants sous la forme exponentielle :

1. $1 = e^0$ car $r = \sqrt{1^2+0^2} = 1$ et $\cos^{-1}\left(\frac{1}{1}\right) = 0$.

2. $5 = 5e^0$ car $r = \sqrt{5^2+0^2} = 5$ et $\cos^{-1}\left(\frac{5}{5}\right) = 0$.

3. $i = e^{\frac{\pi}{2}i}$ car $r = \sqrt{0^2+1^2} = 1$ et $\cos^{-1}\left(\frac{0}{1}\right) = \frac{\pi}{2}$.

4. $-\sqrt{2} = \sqrt{2}e^{\pi i}$ car $r = \sqrt{(-\sqrt{2})^2+0^2} = \sqrt{2}$ et $\cos^{-1}\left(\frac{-\sqrt{2}}{\sqrt{2}}\right) = \pi$.

5. $-3i = 3e^{-\frac{\pi}{2}i}$ car $r = \sqrt{0^2+(-3)^2} = 3$ et $\cos^{-1}\left(\frac{0}{3}\right) = -\frac{\pi}{2}$.

6. $1+i = \sqrt{2}e^{\frac{\pi}{4}i}$ car $r = \sqrt{1^2+1^2} = \sqrt{2}$ et $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.

7. $1-i = \sqrt{2}e^{-\frac{\pi}{4}i}$ car $r = \sqrt{1^2+(-1)^2} = \sqrt{2}$ et $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$.

8. $-3+3i = 3\sqrt{2}e^{\frac{5\pi}{4}i}$ car $r = \sqrt{(-3)^2+3^2} = 3\sqrt{2}$ et $\cos^{-1}\left(\frac{-3}{3\sqrt{2}}\right) = \frac{5\pi}{4}$.

9. $-2-2i = 2\sqrt{2}e^{\frac{3\pi}{4}i}$ car $r = \sqrt{(-2)^2+(-2)^2} = 2\sqrt{2}$ et $\cos^{-1}\left(\frac{-2}{2\sqrt{2}}\right) = \frac{3\pi}{4}$.

10. $\frac{\sqrt{3}}{2} + \frac{i}{2} = e^{\frac{\pi}{6}i}$ car $r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$ et $\cos^{-1}\left(\frac{\sqrt{3}/2}{1}\right) = \frac{\pi}{6}$.

11. $\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = e^{\frac{5\pi}{4}i}$ car $r = \sqrt{\left(\frac{-\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = 1$ et $\cos^{-1}\left(\frac{-\sqrt{2}/2}{\sqrt{2}}\right) = \frac{5\pi}{4}$.

12. $\frac{1}{2} - \frac{\sqrt{3}}{2}i = e^{-\frac{\pi}{3}i}$ car $r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = 1$ et $\cos^{-1}\left(\frac{1/2}{1}\right) = -\frac{\pi}{3}$.

13. $3-\sqrt{3}i = 2\sqrt{3}e^{-\frac{\pi}{6}i}$ car $r = \sqrt{3^2+(-\sqrt{3})^2} = 2\sqrt{3}$ et $\cos^{-1}\left(\frac{3}{2\sqrt{3}}\right) = -\frac{\pi}{6}$.

14. $-5\sqrt{3}+5\sqrt{3}i = 10\sqrt{3}e^{\frac{5\pi}{3}i}$ car $r = \sqrt{(-5\sqrt{3})^2+(5\sqrt{3})^2} = 10\sqrt{3}$ et $\cos^{-1}\left(\dfrac{-5\sqrt{3}}{10\sqrt{3}}\right) = \frac{5\pi}{3}$.

15. $\sqrt{2}+\frac{\sqrt{6}}{3}i = \frac{2\sqrt{6}}{3}e^{\frac{\pi}{4}i}$ car $r = \sqrt{\left(\sqrt{2}\right)^2+\left(\frac{\sqrt{6}}{3}\right)^2} = \frac{2\sqrt{6}}{3}$ et $\cos^{-1}\left(\dfrac{\sqrt{2}}{\frac{2\sqrt{6}}{3}}\right) = \frac{\pi}{4}$.

16. $-\pi\sqrt{3}-\pi i = \pi\sqrt{3}e^{-\pi i}$ car $r = \sqrt{(-\pi\sqrt{3})^2+(-\pi)^2} = \pi\sqrt{3}$ et $\cos^{-1}\left(\dfrac{-\pi\sqrt{3}}{\pi\sqrt{3}}\right) = -\pi$.